

Applications of the Fundamental Theorem of Calculus

Calculator Exercises

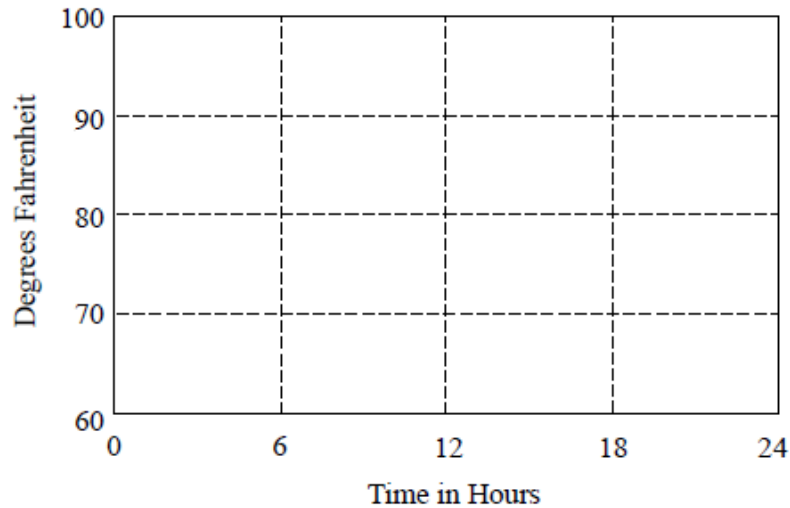
1.

The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10 \cos\left(\frac{\pi t}{12}\right), \quad 0 \leq t \leq 24,$$

where  $F(t)$  is measured in degrees Fahrenheit and  $t$  is measured in hours.

(a) Sketch the graph of  $F$  on the grid below.



- (b) Find the average temperature, to the nearest degree Fahrenheit, between  $t = 6$  and  $t = 14$ .
- (c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of  $t$  was the air conditioner cooling the house?
- (d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

2.

The number of gallons,  $P(t)$ , of a pollutant in a lake changes at the rate  $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$  gallons per day, where  $t$  is measured in days. There are 50 gallons of the pollutant in the lake at time  $t = 0$ . The lake is considered to be safe when it contains 40 gallons or less of pollutant.

- (a) Is the amount of pollutant increasing at time  $t = 9$ ? Why or why not?
- (b) For what value of  $t$  will the number of gallons of pollutant be at its minimum? Justify your answer.
- (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
- (d) An investigator uses the tangent line approximation to  $P(t)$  at  $t = 0$  as a model for the amount of pollutant in the lake. At what time  $t$  does this model predict that the lake becomes safe?

3.

The rate at which people enter an amusement park on a given day is modeled by the function  $E$  defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function  $L$  defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both  $E(t)$  and  $L(t)$  are measured in people per hour and time  $t$  is measured in hours after midnight. These functions are valid for  $9 \leq t \leq 23$ , the hours during which the park is open. At time  $t = 9$ , there are no people in the park.

- How many people have entered the park by 5:00 P.M. ( $t = 17$ )? Round your answer to the nearest whole number.
- The price of admission to the park is \$15 until 5:00 P.M. ( $t = 17$ ). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
- Let  $H(t) = \int_9^t (E(x) - L(x)) dx$  for  $9 \leq t \leq 23$ . The value of  $H(17)$  to the nearest whole number is 3725. Find the value of  $H'(17)$ , and explain the meaning of  $H(17)$  and  $H'(17)$  in the context of the amusement park.
- At what time  $t$ , for  $9 \leq t \leq 23$ , does the model predict that the number of people in the park is a maximum?

4.

A tank contains 125 gallons of heating oil at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- (a) How many gallons of heating oil are pumped into the tank during the time interval  $0 \leq t \leq 12$  hours?
- (b) Is the level of heating oil in the tank rising or falling at time  $t = 6$  hours? Give a reason for your answer.
- (c) How many gallons of heating oil are in the tank at time  $t = 12$  hours?
- (d) At what time  $t$ , for  $0 \leq t \leq 12$ , is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

5.

For  $0 \leq t \leq 31$ , the rate of change of the number of mosquitoes on Tropical Island at time  $t$  days is modeled by  $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$  mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time  $t = 0$ .

- (a) Show that the number of mosquitoes is increasing at time  $t = 6$ .
- (b) At time  $t = 6$ , is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
- (c) According to the model, how many mosquitoes will be on the island at time  $t = 31$ ? Round your answer to the nearest whole number.
- (d) To the nearest whole number, what is the maximum number of mosquitoes for  $0 \leq t \leq 31$ ? Show the analysis that leads to your conclusion.

6.

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function  $F$  defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \leq t \leq 30,$$

where  $F(t)$  is measured in cars per minute and  $t$  is measured in minutes.

- (a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
- (b) Is the traffic flow increasing or decreasing at  $t = 7$ ? Give a reason for your answer.
- (c) What is the average value of the traffic flow over the time interval  $10 \leq t \leq 15$ ? Indicate units of measure.
- (d) What is the average rate of change of the traffic flow over the time interval  $10 \leq t \leq 15$ ? Indicate units of measure.